Honors Advanced Math

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Review 1 - Answers

1. Look at the graph of each function to determine whether the function is even or odd. Even functions will have y-axis reflection symmetry. Odd functions will have origin symmetry (180 degree origin rotational symmetry).

a. $f(-x) = 3\tan(-x) + 2\sin(-x) = 3(-\tan x) + 2(-\sin(x)) = -3\tan x - 2\sin x$

$\therefore\ f(-x) = -f(x)$, so $f(x)$ is odd.

b. $f(-x) = \frac{2(-x)^2}{4-(-x)^2} = \frac{2x^2}{4-x^2}$.

$\therefore\ f(x) = f(-x)$, so f is even.

c. $f(-x) = (-x)^3 + 5 = -x^3 + 5$.

$\therefore\ f(x) \neq f(-x),\ f(-x) \neq -f(x)$, so f is neither even nor odd.

2. Describe a sequence of graphical transformations to turn the graph of $f(x)$ into the graph of $g(x)$.

a. $f(x) = 2x^2 + 1,\ g(x) = -2(x+7)^2 + 5$

1. shift left by 7 units.
2. flip over x-axis.
3. shift up by 6 units.

b. either of these methods will work:

way 1: 1. shift left by 5. 2. shrink horizontally by a factor of 2 (half as wide).

way 2: 1. shrink horizontally by a factor of 2 (half as wide). 2. shift left by 5/2.

3. $(g \circ f)(x) = g(f(x)) = g(x^2+5) = \sqrt{x^2+5}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 5 = x + 5$

4. Suppose $g(x) = \frac{2x+3}{x-5}$. Find $\lim_{x\to\infty} g^{-1}(x)$.

One method: Calculate that $\boldsymbol{g}^{-1}(\boldsymbol{x}) = \frac{3+5\boldsymbol{x}}{\boldsymbol{x}-2}$, which has a horizontal asymptote at y = 5, so

$\lim_{x\to\infty} g^{-1}(x) = 5$.

Another method: $g(x)$ has a vertical asymptote at $x = 5$, so $g^{-1}(x)$ has a horizontal asymptote at $y = 5$, so $\lim_{x\to\infty} g^{-1}(x)$

5. Find values of b and c that make $f(x)$ continuous on its domain.

Continuity at $x = -1$ requires that $1 - b + c = 10$; continuity at $x = 2$ requires that $4 + 2b + c = 1$. Solve the linear system (substitution, row reduction, inverse matrix) to get
$b = -4$ and $c = 5$.

Review 1 - Answers

6. *C* must have 3 zeros (it is a cubic). Since $1-2i$ is a zero, $1+2i$ must also be a zero (real coefficients, complex conjugates theorem). C must have the form:

$C(x)=a(x-2)(x-(1-2i))(x-(1+2i))$, where a will be determined by the condition $f(1)=-12$.

$C(x)=a(x^3-4x^2+9x-10)$

$C(1)=a(1-4+9-10)=-12 \Rightarrow a=3$

7. $\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di}=\frac{(ac+bd)+(cb-ad)i}{c^2+d^2}=(\frac{ac+db}{c^2+d^2})+(\frac{cb-ad}{c^2+d^2})i$

the last expression can be though of as: (real number) + (real number)i.

8. **a.** $x^3=x^2+23x+42$

$x^3-x^2-23x-42=0$;Graph on calc and find that that $x=6$ is a rational zero, so factor out (x-6) to get:

$(x-6)(x^2+5x+7)=0$ use the quadratic formula to find the other two solutions:

solutions: $x=6,\ \frac{-5\pm\sqrt{3}i}{2}$.

b. $x^3=-8$

same method can be used as in 8a. solutions are: $x=-2, 1\pm\sqrt{3}i$. this problem can also be seen as "find the cube roots of -8." nth roots of complex numbers are covered in review 4.

9. Suppose $f(x)=\begin{cases}\cos x & if \quad x<\pi \\ \frac{2x^2-7x-15}{x^2-9x+20} & if \quad x\ge\pi\end{cases}$. Find

a. $\lim_{x\to\pi} f(x)=$ does not exist (left and right limits don't agree)

b. $\lim_{x\to\pi^-} f(x)=-1$

c. $\lim_{x\to 4^-} f(x)=-\infty$

d. $\lim_{x\to 5} f(x)=13$ **e.** $\lim_{x\to -\infty} f(x)=$ does not exist (oscillates repeatedly between -1 and 1)

f. $\lim_{x\to\infty} f(x)=2$ (horizontal asymptote)

10. Hole at $x = -1$
so $(x + 1)$ in numerator <u>and</u> denominator

Zeros at $x = -1, 3$
so $(x + 1)(x - 3)$ in numerator

Asymptotes at $x = -3, 2, 4$ so $(x + 3)(x - 2)(x - 4)$ in denominator

Same direction on both sides of $x = 2$
so multiplicity of 2

y – intercept at $y = 1$ so adjust leading coefficient

$$r(0) = \frac{a(x-3)(x+1)^2}{(x+3)(x+1)(x-2)^2(x-4)} = 1 \quad \text{so } a = 16$$

ANSWER: $r(x) = \dfrac{16(x-3)(x+1)^2}{(x+3)(x+1)(x-2)^2(x-4)}$

11. A polynomial with the right zeros and end behavior:

$$f(x) = (x-2)^2(x+6)(x-(4-i))(x-(4+i)) = (x-2)^2(x+6)(x^2-8x+17)$$

Scaling f so that it passes through (1,2) gives:

$$f(x) = \frac{1}{35}(x-2)^2(x+6)(x^2-8x+17)$$