Honors Advanced Math

Name______________________

Review 2 Answers

1. **a.** A right triangle with angle $\cos^{-1}(\frac{3}{5})$ may have sides of 3, 4, and 5. This gives $\tan(\cos^{-1}(\frac{3}{5})) = \frac{4}{3}$.

b. A right triangle with angle $\cos^{-1}(x)$ may have sides of x, $\sqrt{1-x^2}$, and 1. This gives $\tan(\cos^{-1} x) = \sqrt{1-x^2}\, / \, x$.

2. $\cos(\frac{\pi}{2} - \theta) = \sin\theta$ because the x-coordinate at angle $\frac{\pi}{2} - \theta$ is the same as the y-coordinate at angle θ.

$\sin(\pi - \theta) = \sin\theta$ because the y-coordinates at the same at angle $\pi - \theta$ and angle θ.

Therefore, $\cos(\frac{\pi}{2} - \theta) = \sin(\pi - \theta)$.

3. **a.** $f(x) = 2.5\sin(\frac{2\pi}{4}(x-2)) + 2.5$ or $-2.5\cos(\frac{2\pi}{4}(x-1)) + 2.5$

b. $g(x) = 2.5\sin(\frac{2\pi}{4}(8x-2)) + 2.5$

4. $\angle AOP = \angle POB = \tan^{-1}(\frac{35}{15}) \approx 1.166$. $\angle AOB \approx 2.332$.
Length arc AB $\approx \frac{2.332}{2\pi} \cdot 2\pi 15 \approx 34.977$. Perimeter $\approx 35 + 35 + 34.977 =$ <u>104.977 units</u>.
Area sector AOB $\approx \frac{2.332}{2\pi} \cdot \pi 15^2 \approx 262.329$.
Shaded area = $\Delta AOP + \Delta BOP$ – sector AOB = 262.5 + 262.5 – 262.329 $\approx$ <u>262.671 sq. units</u>.

5. Law of Sines gives $\sin\angle T = \frac{8\sin 45°}{10} \approx 0.566$; $\angle T \approx 34.45°$ (or 145.55°, but that value is impossible because of the 180° angle sum).

Next, $\angle S \approx 100.55°$. Finally, Law of Cosines using $\angle S$ gives that RT = 13.903.

6. Law of Cosines on ΔXYZ gives XZ $\approx$ 7.82.
Law of Sines on ΔXYZ gives $\angle YZX \approx 78.40°$, then $\angle XZW \approx 31.60°$.
Law of Cosines on ΔXWZ gives <u>XW $\approx$ 4.15</u>.
Law of Cosines on ΔXWZ gives <u>$\angle W \approx 99.16°$</u>, then <u>$\angle X \approx 100.84°$</u>.

Area = $\frac{1}{2} \cdot 10 \cdot 8 \cdot \sin(50°) + \frac{1}{2} \cdot 7.82 \cdot 6 \cdot \sin(31.60°) \approx$ <u>42.93 sq. units</u>.

7. a. $3x = \frac{7\pi}{6} + 2n\pi$ or $3x = \frac{11\pi}{6} + 2n\pi$
$x = \frac{7\pi}{18} + \frac{2}{3}n\pi$ or $x = \frac{11\pi}{18} + \frac{2}{3}n\pi$
Solutions in the specified interval: $\{\frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}\}$.

b. $2\cos(3x-1) = \sqrt{3}$

$$\cos(3x-1) = \frac{\sqrt{3}}{2}$$

$$(3x-1) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} + 2\pi n, \quad \frac{11\pi}{6} + 2\pi n$$

$$x = \frac{\pi}{18} + \frac{2\pi n}{3} + \frac{1}{3}, \quad \frac{11\pi}{18} + \frac{2\pi n}{3} + \frac{1}{3}$$

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8. $\sin^2 x - \sin x = 1 - \sin^2 x$
$2\sin^2 x - \sin x - 1 = 0$
$(2\sin x + 1)(\sin x - 1) = 0$
$\sin x = -\frac{1}{2}$ or $\sin x = 1$
Solutions in the specified interval: $\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$.

9. **a.** $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{4}{5}\cdot\frac{-7}{25} - \frac{3}{5}\cdot\frac{24}{25} = \frac{-100}{125} = \frac{-4}{5}$.

b.

$$\tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$$

$$\tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$$

10.

$$\cos(2x) = \cos(x + x)$$
$$= \cos x \cdot \cos x - \sin x \cdot \sin x$$
$$= \cos^2 x - \sin^2 x$$
$$= (1 - \sin^2 x) + \sin^2 x$$
$$= 1 - 2\sin^2 x$$

Equating the first and last expressions in this chain and solving for $\sin^2 x$ gives:

$$\sin^2 x = \frac{\cos(2x) - 1}{-2} = \frac{1 - \cos(2x)}{2}$$

11.
$\cos^2(x) + \cos(x) = 1 - \cos^2(x)$

$2\cos^2(x) + \cos(x) - 1 = 0$

$(2\cos(x) - 1)(\cos(x) + 1) = 0$

$\cos(x) = 1/2$ or $\cos(x) = -1$

So $x = \frac{\pi}{3} + 2\pi n$, $\frac{5\pi}{3} + 2\pi n$, $\pi + 2\pi n$ for all integers n.

12. a. $D(t) = 5\sin(\frac{2\pi}{12.4}(t - 8.5)) + 25$.

b. Graphically solve $28 = 5\sin(\frac{2\pi}{12.4}(t - 8.5)) + 25$.

The first two positive solutions are at $t = 1.03$ and $t = 9.77$.

General solution: $t = 1.03 + 12.4n$ and $t = 9.77 + 12.4n$ for all integers