Honors Advanced Math

Name______________________

Review 3 Answers

1a. resulting vector should be <8,1/2>, graphically, 8 units right and 1/2 unit up.

b. $\cos\theta = \frac{v\cdot w}{|v|\cdot|w|} = \frac{-21}{15\sqrt{29}} \Rightarrow \theta \approx 105°$

c. Twice the length of a unit vector in the direction of **v**: $2\cdot\frac{<12,-9>}{15} =< \frac{8}{5},-\frac{6}{5} >$

d. $(-7,1)+\frac{2}{5}\cdot<12,-9> = (-\frac{11}{5},-\frac{13}{5})$

2. the vector equation of the line would be: $(x,y) = (0,2)+<3,-2>t$. This yields paramtric equations $\begin{cases} x(t) = 0+3t \\ y(t) = 2-2t \end{cases}$. This will be a ray on the interval: $0 \le t < \infty$.

3a. Vector: $(x,y,z) = \langle 2,-1,-3\rangle t + (-7,2,-1)$. Parametric: $x = 2t-7$, $y = -t+2$, $z = -3t-1$. Other answers are possible.

b. $3x-2y+z = d \to 3(4)-2(8)-1 = -5 \to 3x-2y+z = -5$

c. $3(2t-7)-2(-t+2)+-3t-1 = -5 \to t = \frac{21}{5} \to \left(\frac{7}{5},-\frac{11}{5},-\frac{68}{5}\right)$

4a. $\overrightarrow{AB} =< 6,-2,5 >$, $\overrightarrow{AC} =< 7,1,7 >$, $\left|\overrightarrow{AB}\right| = \sqrt{65}$, $\left|\overrightarrow{AC}\right| = \sqrt{99}$. Using the dot product we get: $\overrightarrow{AB}\cdot\overrightarrow{AC} = (6)(7)+(-2)(1)+(5)(7) = \sqrt{65}\sqrt{99}\cos\theta$, which yields $\theta \approx 20.78^\circ$.

b. $AREA = \frac{1}{2}\sqrt{65}\sqrt{99}\sin 20.78^\circ = 14.23°$

c. $-19x-7y+20x = -42$

5.
$$l_1 : (x,y,z) = \langle -2,-4,1\rangle t + (1,6,5)$$
$$l_2 : (x,y,z) = \langle 1,0,-5\rangle s + (-2,-2,2)$$

a. Equate the *x*, *y*, and *z* coordinates to get this system:

$$-2t+1 = s-2$$
$$-4t+6 = -2$$
$$t+5 = -5s+2$$

The system's solution is $t = 2$, $s = -1$, giving intersection point $(x, y, z) = (-3, -2, 7)$.

b. Let θ denote the angle between the lines' direction vectors.

$$\cos\theta = \frac{< -2,-4,1 > \cdot < 1,0,-5 >}{|< -2,-4,1 >|\ |< 1,0,-5 >|} = \frac{-7}{\sqrt{21}\sqrt{26}};\ \theta = \cos^{-1}\left(\frac{-7}{\sqrt{21}\sqrt{26}}\right) \approx 1.875 \text{ radians..}$$

It turns out that θ is the obtuse angle between the lines, so the acute angle is its supplement: $\pi - 1.875 \approx 1.267$ radians.

c. First get a vector perpendicular to the plane by calculating the cross product of the lines' direction vectors:

Honors Advanced Math

Name________________________

Review 3 Answers

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -4 & 1 \\ 1 & 0 & -5 \end{vmatrix} = 20\mathbf{i} - 9\mathbf{j} + 4\mathbf{k}.$$

The plane's equation must be of the form $20x - 9y + 4z = D$. Substitute any point on the plane (such as the intersection found in part **a**) to get $D = -14$.
Answer: $20x - 9y + 4z = -14$.

6. $P_1 : 2x + 2y + z = -2$
$P_2 : 3x + 2z = -2$
$P_3 : -x + 5y - 2z = 1$

a. $\left[\begin{array}{ccc|c} 2 & 2 & 1 & -2 \\ 3 & 0 & 2 & -2 \\ -1 & 5 & -2 & 1 \end{array}\right]$ row reduces to $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -4 \end{array}\right]$ so the intersection is point $(2, -1, -4)$.

b. $\left[\begin{array}{ccc|c} 2 & 2 & 1 & -2 \\ 3 & 0 & 2 & -2 \end{array}\right]$ row reduces to $\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & -\frac{2}{3} \\ 0 & 1 & -\frac{1}{6} & -\frac{1}{3} \end{array}\right]$.

The reduced matrix represents the system $x + \frac{2}{3}z = -\frac{2}{3}$, $y - \frac{1}{6}z = -\frac{1}{3}$.
Let $z = t$ and solve the equations for x and y: $x = -\frac{2}{3}t - \frac{2}{3}$, $y = \frac{1}{6}t - \frac{1}{3}$.
Finally write the three parametric equations as a vector equation:
$(x, y, z) = \langle -\frac{2}{3}, \frac{1}{6}, 1\rangle\, t + (-\frac{2}{3}, -\frac{1}{3}, 0)$.

7. a. $(1,0) \to (a, b)$; $(0,1) \to (c, d)$.

b. True: $x' = 0x + 0y = 0$ and $y' = 0x + 0y = 0$.

8. a. $x' = -y$, $y' = -x$.

b. $x' = (\cos\theta)x + (\sin\theta)y$, $y' = (-\sin\theta)x + (\cos\theta)y$.

9. a. $\begin{bmatrix} \cos 60° & -\sin 60° \\ \sin 60° & \cos 60° \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1.598 \\ 3.232 \end{bmatrix}$.

b. $\begin{bmatrix} \cos 120° & -\sin 120° \\ \sin 120° & \cos 120° \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$;

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 120° & -\sin 120° \\ \sin 120° & \cos 120° \end{bmatrix}^{-1} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \approx \begin{bmatrix} 0.598 \\ -4.964 \end{bmatrix}$.

10. Substitute each of the 4 given points in place of x and y in the equation $y = ax^3 + bx^2 + cx + d$.
This gives a system of four equations in four variables. Solve using rref to get a, b, c, and d.
Answer: $f(x) = 0.25x^3 + 0x^2 - 1x + 3$.

Honors Advanced Math

Name________________________

Review 3 Answers

11. Subtract $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ from both sides; multiply both sides by –1; left-multiply both sides by the inverse of the 2-by-2 matrix. Answer: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 13/8 \end{bmatrix}$.

12. Setup is $\frac{3x+4}{x^2+14x+48} = \frac{A}{x+6} + \frac{B}{x+8}$. Multiply both sides by $(x + 6)(x + 8)$ to get $3x + 4 = A(x + 8) + B(x + 6) = (A + B)x + (8A + 6B)$.

Equate coefficients to get the system $3 = A + B$, $4 = 8A + 6B$. Solution is $A = –7$, $B = 10$.

Answer: $\frac{3x+4}{x^2+14x+48} = \frac{-7}{x+6} + \frac{10}{x+8}$.