Honors Advanced Math

Name________________________

Review 4 Answers

1. Both limits can be found graphically:

a. $\lim_{x\to\infty} \tan^{-1} x = \pi/2$ b. $\lim_{x\to 2+} \ln(x-2) = -\infty$

b. Find a pair of real numbers r and s such that $2\ln(5) = \dfrac{\log_4 r}{\log_4 s}$.

Left side is: $2\ln(5) = \ln 5^2 = \ln 25 = \dfrac{\ln 25}{\ln e}$

Right side is: $\dfrac{\log_4 r}{\log_4 s} = \dfrac{\ln r/\ln 4}{\ln s/\ln 4} = \dfrac{\ln r}{\ln s}$.

So $r = 25$ and $s = e$ is a possible answer. There are other answers also.

2. **a.** Let $L = \log_b 3$, $M = \log_b 4$, and $N = \log_b 5$. Express $\log_b 30$ in terms of some combination of L, M, and N.

Solution: $M = \log_b 4 \Rightarrow M = 2\log_b 2 \Rightarrow \frac{1}{2}\boldsymbol{M} = \mathbf{log_b} 2$.

$\mathbf{log_b} 30 = \mathbf{log_b}(3\cdot 2\cdot 5) = \mathbf{log_b} 3 + \mathbf{log_b} 2 + \mathbf{log_b} 5 = \boldsymbol{L} + \frac{1}{2}\boldsymbol{M} + \boldsymbol{N}$

b. Prove the identity $\log_b uv = \log_b u + \log_b v$. You may only assume the definition of logarithm (e.g. $\log_b x = y \Leftrightarrow b^y = x$) in your proof.

(*Hint to get started: Let* $p = \log_b u$ *and* $q = \log_b v$)

Solution: $\boldsymbol{p} = \mathbf{log_b}\,\boldsymbol{u} \Rightarrow b^p = u$ and $q = \log_b v \Rightarrow b^q = v$.

Therefore, $\boldsymbol{b^p \cdot b^q = uv} \Rightarrow b^{p+q} = uv \Rightarrow \log_b uv = p + q = \log_b u + \log_b v$

3. **a.** $z = 1 + i = \sqrt{2}cis45^\circ$. $z^8 = (\sqrt{2}cis45^\circ)^8 = 16cis360 = 16$. $|z| = \left|\sqrt{2}cis45^\circ\right| = \sqrt{2}$.

b. one of the 4th roots: $\sqrt[4]{\sqrt{2}cis(\pi/4)} = \sqrt[8]{2}cis(\pi/16)$ the other 3 are equally spaced around a circle of radius $\sqrt[8]{2}$. The other 3 would be: $\sqrt[8]{2}cis(\pi/16 + \pi/2)$, $\sqrt[8]{2}cis(\pi/16 + \pi)$, $\sqrt[8]{2}cis(\pi/16 + 3\pi/2)$.

4. This is equivalent to the equation $x^5 = 32$, which is equivalent to finding the five 5th roots of 32. one of the 5th roots of 32 is 2. the other 4 are equally spaced around a circle of radius 2. Concisely, the 5 solutions are: $2cis(\frac{360}{5}n)$, where $n = 0,1,2,3,4$.

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5.

cartesian	Convert	polar
$(-\frac{1}{2},\frac{\sqrt{3}}{2})$	→	$(1,\frac{2\pi}{3}) = (1,\ 120°)$
$(4\cos(\frac{5\pi}{6}),4\sin(\frac{5\pi}{6}))$ $=(-2\sqrt{3},2)$	←	$(4,\frac{5\pi}{6})$
$x^2+y^2=12$	→	$r=\sqrt{12}$
$y=4$	←	$r=4\csc\theta$

6. Let x = radium mass 5,000 years ago. Solve $x \cdot (\frac{1}{2})^{5000/1600} = 200$ to get $x \approx 1744.81$ mg.

7. $\log_3 450 = \log_3(2\cdot3\cdot3\cdot5\cdot5) = \log_3 2 + \log_3 3 + \log_3 3 + \log_3 5 + \log_3 5 = a + 2 + 2b$.

8. $g(x) = (\frac{1}{8})^x = (2^{-3})^x = ((4^{1/2})^{-3})^x = 4^{-\frac{3}{2}x} = f(-\frac{3}{2}x)$, so the transformations are:

- reflection across the y-axis
- horizontal shrink by a factor of 3/2

done in either order.

9. **a.** Domain is all real numbers x such that $x > 2$ or $x < -2$. Range is all real numbers.

b. $\lim_{x\to(-2)-} F(x) = -\infty$ and $\lim_{x\to 2+} F(x) = -\infty$.

10. Applying the change-of-base formula: $\log_b c = \frac{\log_c c}{\log_c b} = \frac{1}{\log_c b}$

11. a. $(r \sin\theta) = 2(r\cos\theta) + 3$. Solve for r to get $r = \frac{3}{\sin\theta - 2\cos\theta}$.

b. $r = \frac{1}{\frac{x}{r}+\frac{y}{r}} \Rightarrow r = \frac{r}{x+y} \Rightarrow x + y = 1 \Rightarrow y = 1 - x$.

c. Curve is a circle with center (2, 0) and radius = 2.

$$r = 4\cos\theta$$
$$r^2 = 4r\cos\theta$$
$$x^2 + y^2 = 4x$$
$$(x-2)^2 + y^2 = 4$$