Honors Math 4 Name:________________________________
Conic Section Review Problems 2 June 2017

Directions: Here are some extra problems on conic sections. Some are problem types that we've already done and some are a little new. All of them are within your reach. In studying for your test, you should also make sure you understand the problems from the previous homework assignments and that you know how to do all of the major proofs from the unit.

1. Consider the equation $\frac{(x+3)^2}{4} - \frac{(y+7)^2}{9} = 1$.

a. Sketch the graph without using your calculator. Include the coordinates of all major points (center, vertices, foci), any asymptotes (with equations for the asymptotes), and the directrices (with equations – you will need to determine the eccentricity first).

b. Asymptotes can be useful for estimating the coordinates of points on a curve. Use the asymptotes that you got in part **a** to estimate the values of y when x = 347. Then check your estimation by plugging x = 347 into the equation $\frac{(x+3)^2}{4} - \frac{(y+7)^2}{9} = 1$ and solving for y. You may use your calculator for the second part of this problem.

c. List a sequence of graphical transformations that would turn the graph $y^2 - x^2 = 1$ into the graph of $\frac{(x+3)^2}{4} - \frac{(y+7)^2}{9} = 1$. Note: There isn't a typo. You can figure this out.

2. A tangent line of a parabola is a line that intersects but does not cross the parabola. Prove that a line tangent to the parabola $x^2 = 4py$ at the point (a, b) crosses the y-axis at (0, -b).
(*Hints:* Draw and label a diagram. Write equations for the parabola and the line. Use substitution to combine the equations. You should end up with a quadratic equation. This equation should have only one solution because the parabola and line intersect once. Quadratic equations with only one solution have a discriminant of zero.)

3. Consider the ellipse $\frac{(x-1)^2}{25} + \frac{(y+4)^2}{9} = 1$.

a. Write parametric equations for x and y that would graph this ellipse. Set up your equations so that $t = 0$ yields the point $(6,-4)$, the graph goes counterclockwise, and the graph first returns to the point $(6,-4)$ when $t = 2\pi$.

b. Write a different set of parametric equations for x and y that would graph this ellipse. This time, set up your equations so that the graph moves in a clockwise direction. Set it up so that $t = 0$ will still yields the point $(6,-4)$, but the graph first returns to the point $(6,-4)$ when $t = \pi$.

c. Consider the parametric equations $x = 5\sin t + 1$ and $y = 3\cos t - 4$. Will these equations graph the ellipse $\frac{(x-1)^2}{25} + \frac{(y+4)^2}{9} = 1$? How are these parametric equations different from the equations in parts a and b?

4. Consider the hyperbola $\frac{(x+3)^2}{4} - 9y^2 = 1$. Write a pair of a parametric equations for x and y that would graph this hyperbola. Then sketch the graph. On your sketch, you should include arrows indicating direction and an indication of the t values where your graph begins and ends. Your graph should also include the coordinates of the important points and the asymptotes with their equations.

5. Consider the parametric equations $x = \frac{1}{2}\tan t + 1$ and $y = \frac{1}{3}\sec t + 8$.

a. Sketch the graph. On your sketch, you should include arrows indicating direction and an indication of the t values where your graph begins and ends. Your graph should also include the coordinates of the important points and the asymptotes with their equations.

b. Write a Cartesian equation for the curve described by $x = \frac{1}{2}\tan t + 1$ and $y = \frac{1}{3}\sec t + 8$.

c. Determine the eccentricity of the shape and write the equations of the directrices.

6. An ellipse E is created in the following way:

- Start with the unit circle $x^2 + y^2 = 1$
- Stretch horizontally be a factor of 3 and compress vertically by a factor of 2
- Shift left by 4 and down by 3

a. Write the Cartesian equation for E.

b. Write a parametric equation for E. Set up your equations so that at time $t = 0$, the ellipse is at its highest point, it rotates counterclockwise, and it does one full period every 4π.

c. Sketch the ellipse. Include the coordinates of the center, the vertices, the foci, and the equations of the directrices.

d. Shift the ellipse so that one focus is at the origin. Rewrite the Cartesian equation for E. Write the polar equation for E.

7. A hyperbola H is created in the following way:

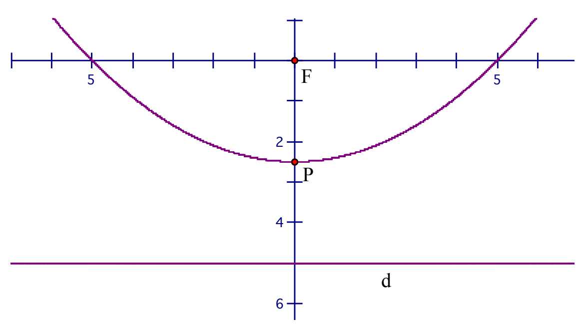
- Start with the unit hyperbola $x^2 - y^2 = 1$
- Reflect over the line $y = x$
- Stretch horizontally be a factor of 2 and stretch vertically by a factor of 5
- Shift left by 2 and down by 7

a. Write the Cartesian equation for H.

b. Write a parametric equation for H. Set up your equations so that at time $t = 0$, the hyperbola is at a point directly above its center, and it goes through one full period every $\frac{\pi}{2}$. (It can move in either direction).

c. Sketch the hyperbola. Include the coordinates of the center, the vertices, the foci, the equations of the asymptotes and the equations of the directrices.

d. Shift the hyperbola so that one focus is at the origin. Rewrite the Cartesian equation for H. Write the polar equation for H.



8. At right is a graph of a conic section with it's focus at the origin and its directrix at $y = -5$. Since the graph is cut off it is not clear if the conic section is an ellipse, an hyperbola or a parabola. All that can be seen is a point on the curve at (0, –2.5).

a. From the given information, determine the eccentricity, *e*. (Remember to show work or justify your answer!) What kind of conic section is it?

b. Write a polar equation for this curve.

c. On the graph above, indicate the points where $\theta = 0$ and $\theta = \frac{3\pi}{4}$.

d. Write a new polar equation which rotates this curve <u>clockwise</u> by $\frac{7\pi}{5}$.

e. Write the polar equation of the directrix for the rotated conic. (*Hint:* write the polar equation for the original directrix first.)