Honors Advanced Math
Extension Problems

Name:_____
Block: E or G 12 February 2014

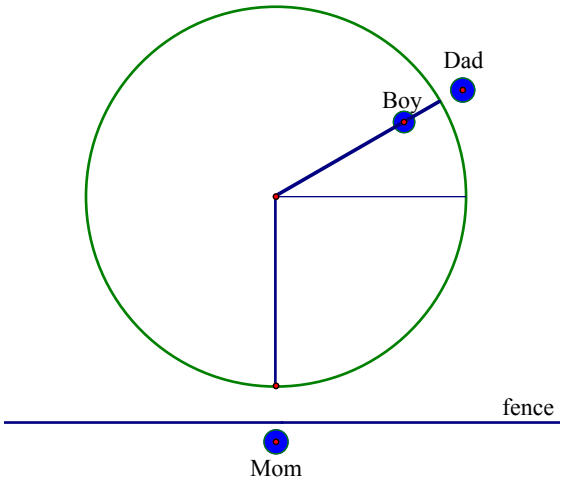
1. ***Hero's Formula*** The goal of this problem is to prove Hero's formula (also sometimes known as Heron's formula). The formula states that the area of any triangle ABC is given by $\sqrt{s(s-a)(s-b)(s-c)}$, where *a*, *b*, and *c* are the sides lengths and *s* is the semi-perimeter, defined as $s = \frac{a+b+c}{2}$.

   a. Rewrite the expression $\sin\left(\cos^{-1}(x)\right)$ without using any trigonometric functions.

   b. Show that for any triangle, $C = \cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right)$

   c. Prove Hero's formula. Hint: Start with $Area = \frac{1}{2}ab\sin C$. Then use the results of part *a* and *b* to turn $\frac{1}{2}ab\sin C$ into an expression involving just *a*, *b*, and *c* with no trigonometric functions and go from there. One more hint: Factor!

2. You are on a viewing platform looking down at a merry-go-round. You see a dad help a little boy get on the merry-go-round at the 2 o'clock position. The little boy's mom is at the 6 o'clock position behind a fence 5 feet from the merry-go-round. The merry-go-round has a radius of 20 feet and takes 30 seconds to go around. The little boy chooses a horse that is two feet from the edge of the merry-go-round.



   a. Sketch a graph of the boy's distance <u>from the fence</u> as a function of the time he spends on the merry-go-round.

b. Find the equation of the boy's distance from the fence versus time using cosine.

c. Rewrite the equation to use sine.

d. Find the times that the boy is nearest to his mom.

e. Let us revisit the merry-go-round problem. Remember, the dad puts the boy on the merry-go-round at the 2 o'clock position and the mom is behind a fence at the 6 o'clock position. This time the merry-go-round has a radius of 30 feet and takes 40 seconds to go around. The boy is 3 feet from the edge and the fence is 2 feet away from the merry-go-round. Find $y(t)$, the distance of the boy from the fence. This is the "vertical" distance of the boy from his mom. Then find $x(t)$, the "horizontal" distance of the boy from his mom. Use $x(t)$ and $y(t)$, and the distance formula, to find a function for $d(t)$, the boy's distance from his mom.