Name:__________________ Honors Advanced Math
June 2013 Final Exam 2010 ANSWERS

2010 Honors Advanced Math Final: ANSWERS

1. $x = \frac{1}{2}\sin^{-1}\left(\frac{5}{6}\right) + n\pi$ or $x = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{5}{6}\right) + n\pi$

2. $\left(2\cos\frac{\pi}{5} - \sin\frac{\pi}{5}, -2\sin\frac{\pi}{5} - \cos\frac{\pi}{5}\right)$ or (1.03, -1.98)

3. $P = \left(5, \pi + \sin^{-1}\left(\frac{4}{5}\right)\right)$ or $P = \left(-5, \sin^{-1}\left(\frac{4}{5}\right)\right)$

4. Sequence is arithmetic:
$\ln 4, \ln 4 + \ln 3, \ln 4 + 2\ln 3, \ln 4 + 3\ln 3, \ldots$

$S_{230} = \frac{n}{2}(t_1 + t_{230}) = 115(\ln 4 + \ln 4 + 229\ln 3) = 115\left(\ln(4^2 3^{229})\right)$

5. $h(t) = 2\cos\left(\frac{\pi}{6}(t+4)\right) + 4$

6. $\sin\left(\tan^{-1}\left(\frac{x}{4}\right)\right) = \frac{x}{\sqrt{x^2+16}}$

7. a. $\vec{v} \cdot \vec{w} = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

b. $\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}|\cos\theta = \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

8. $b = 2.51$

9. a. Zeros: $-5, 11, -\frac{1}{6} + \frac{i\sqrt{59}}{6}, -\frac{1}{6} - \frac{i\sqrt{59}}{6}$ b. Remainder: $P(k) = 3k^4 - 17k^3 - 166k^2 - 85k - 275$

10. a. $y = \log_a c \Leftrightarrow a^y = c$

$\log_b a^y = \log_b c$

$y\log_b a = \log_b c$

$\log_a c \log_b a = \log_b c$

Therefore: $\log_a c = \frac{\log_b c}{\log_b a}$

b. $5 \cdot 4^x = 5\left(3^{\log_3 4}\right)^x = 5 \cdot 3^{\log_3 4x}$ $k = \log_3 4 = 1.262$

11. a. $f(x) = \frac{2x}{x^2 - 4}$; $f(-x) = \frac{1}{-x+2} + \frac{1}{-x-2} = \frac{1}{2-x} - \frac{1}{2+x} = \frac{2x}{4-x^2} = -f(x)$ **Odd**

b. Horizontal stretch by a factor of 5, right shift by 3

13. a. Area = $2\sqrt{6}$

b. If $\mathbf{v}_3$ lies in the plane of $\mathbf{v}_1$ and $\mathbf{v}_2$, then $\mathbf{v}_3$ is perpendicular to the direction vector of the plane, i.e. the dot product is zero: $\langle -2,9,-5\rangle \cdot \langle 8,4,4\rangle = 0$

c. $\frac{5}{4\sqrt{6}}\langle 8,4,4\rangle$

d. Angle is .6126 radians (complement of the angle you get from the dot product of the direction vector and $\mathbf{v}_3$.)

14. a. $f^{-1}(5) = 3$

b. $g^{-1}(x) = f^{-1}\left(e^{x-1}\right)$

c. $\left(1, -\frac{7}{2}\right)$

d. $y = 2f\left(\frac{x}{3}\right)$

15. a. $x_t = 4\cos(t) - 1, \quad y_t = \sqrt{15}\sin(t)$

b. $e = \frac{c}{r_{\max}} = \frac{1}{4}$ and $e = \frac{PF}{Pd}$ so $Pd = 12$ and the equation of the directrix is $x = 15$

c. $\frac{(x-4)^2}{1} - \frac{(y-0)^2}{15} = 1$

d. $e = \frac{c}{r_{trans}} = \frac{4}{1}$, $Pd = 0.75$ and the equation of the directrix is $x = 3.75$

e. $r = \frac{15}{1 + 4\cos\theta}$