

The following is a list of some of the major results that we've proven this year. Any of these could potentially be included as a final exam questions.

- Closure of the complex numbers under division (that is to say, the quotient of two complex numbers is also a complex number)
- $\log_b M + \log_b N = \log_b MN$, $\log_b M - \log_b N = \log_b \frac{M}{N}$, and $\log_b M^c = c \log_b M$
- $\log_a M = \frac{\log_b M}{\log_b a}$
- $\sin^2 \theta + \cos^2 \theta = 1$, $\tan^2 \theta + 1 = \sec^2 \theta$, and $\cot^2 \theta + 1 = \csc^2 \theta$

- Area of triangle = $\frac{1}{2} ab \sin C$
- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$ (see p. 369 of Brown for a reminder)
- $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$ and $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$
- double angle, half angle, and power reducing formulas for sine and cosine (see Demana section 5.4)
- $r_1 \text{cis} \theta_1 \cdot r_2 \text{cis} \theta_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$
- $\frac{r_1 \text{cis} \theta_1}{r_2 \text{cis} \theta_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$
- Equivalence of definitions for dot product (If $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$ then $u \cdot v = u_1 v_1 + u_2 v_2$ and $u \cdot v = \|u\| \|v\| \cos \theta$) (see p. 442 of Brown for a reminder)

Some important notes about this list:

1. **You may be asked to prove results that are not on this list.** Through the year, we have proven many minor results (even/odd proofs, trig identity proofs, etc.) You may see proofs of this type on the final as well. Additionally, please be aware that you may be asked to prove a result that is based on one of these theorems as an immediate consequence. Basically, you should be comfortable with the sort of proof writing expectations that you've had on your unit tests throughout this year.
2. **You will only be held responsible for topics that all sections have covered.** Sequences and Series and Proof by Induction were not covered by all teachers of this class. You do not need to review this topic in preparation for the final.
3. **Do not focus exclusively on this list.** There are many problems on the final and only a few will be proofs. Use the old final exams to give yourself a sense of what to expect.

Formulas you should know for the final...

- $\log_b M + \log_b N = \log_b MN$, $\log_b M - \log_b N = \log_b \frac{M}{N}$, and $\log_b M^c = c \log_b M$
- $\log_a M = \frac{\log_b M}{\log_b a}$
- $\sin^2 \theta + \cos^2 \theta = 1$, $\tan^2 \theta + 1 = \sec^2 \theta$, and $\cot^2 \theta + 1 = \csc^2 \theta$
- $\sec \theta = \frac{1}{\cos \theta}$, $\csc \theta = \frac{1}{\sin \theta}$, $\cot \theta = \frac{1}{\tan \theta}$
- $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$, $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$, $\sin(\pi - \theta) = \sin \theta$, $\cos(\pi - \theta) = -\cos \theta$
- $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$ and $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$
- Area of triangle = $\frac{1}{2} ab \sin C$
- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$
- $r_1 \text{cis} \theta_1 \cdot r_2 \text{cis} \theta_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$
- $\frac{r_1 \text{cis} \theta_1}{r_2 \text{cis} \theta_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$
- Equivalence of definitions for dot product (If $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$ then $u \cdot v = u_1 v_1 + u_2 v_2$ and $u \cdot v = \|u\| \|v\| \cos \theta$) (you should also know the 3-dimensional version of this result.)