Honors Advanced Math Name:__________________________
Review Matrix Problems Answer Key 12 April 2010

1. **a.** $\frac{3}{x+4}+\frac{-2}{x-7}$ **b.** $\frac{-4}{x-5}+\frac{-7}{x}$ **c.** $\frac{4}{x}+\frac{3}{x+2}+\frac{2}{x-5}$

2. **a.** Line, $(x,y,z)=\langle -3,-4,1\rangle t+(5,-2,0)$
b. No solution
c. Point, (2, 3, 0)
d. Line (in 4-space). All points of the form (4, 5, -7, *w*)

3. *k* cannot be 1 or 4. If *k* was equal to either 1 or 4, then the matrix would have a determinant of 0, resulting in either no solutions or infinite solutions.

4. Reflect over the line $y = x$, then stretch by 3 in the x direction and by 4 in the y direction. The matrices are $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ followed by $\begin{bmatrix}3 & 0\\0 & 4\end{bmatrix}$. The order does matter.
Note: There are some other possible answers to this question, but I think this one is the most intuitive. If you have another solution, make sure that the matrices still multiply to $\begin{bmatrix}0 & 3\\4 & 0\end{bmatrix}$ when multiplied in the correct order.

5. $T=\begin{bmatrix}\frac{1}{2} & \frac{-\sqrt{3}}{2}\\ \frac{-\sqrt{3}}{4} & \frac{-1}{4}\end{bmatrix}$. Remember to multiply in the correct order when finding *T*.

6. **a.** The determinant of the matrix is 0 and the matrix is not the zero matrix (which would map everything to the origin)
b. $y = 2x$
c. and d. Any point on the line $3x + y = 4$. This equation is obtained by the knowing that the x-coordinate of the image point is 4. Note: You'd get the equation $6x + 2y = 8$ (the same equation) if you used the fact that the y-coordinate of the image point was 8.
e. All points on the line $3x + y = 4$.
f. The point $(9,6)$ is not on the line $y = 2x$. (And every image point for this transformation is on the line $y = 2x$).

7. **a.** (3, 13, 5) **b.** (1, 6, -3)

8. **a.** Reflection over the *xy*-plane
b. Translation 3 units “forward”, 5 units “left”, 1 unit “down”
c. 90° CCW rotation in the *xz*-plane

9. **a.** $x' = x - 7,\ y' = y - 4,\ z' = z + 10$
b. $x' = -x,\ y' = y,\ z' = z$
c. $x' = -y,\ y' = x,\ z' = z$
d. $x' = x\ ,\ y' = z,\ z' = -y$

10. Linear: $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$ Non-linear: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} h \\ k \\ l \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & h \\ 0 & 1 & 0 & k \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{matrix} x \\ y \\ z \\ 1 \end{matrix} = \begin{matrix} x' \\ y' \\ z' \\ 1 \end{matrix}$

11. **a.** $\begin{bmatrix} a \\ d \\ g \end{bmatrix}$ **b.** $\begin{bmatrix} b \\ e \\ h \end{bmatrix}$ **c.** $\begin{bmatrix} c \\ f \\ i \end{bmatrix}$

12. **a.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **b.** $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ **c.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ **d.** $\begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{bmatrix}$ **e.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$

13. **a.** The determinant of the matrix is 0.
b. Each column of the matrix is an image point, so we know that $(2,1,4)$, $(1,0,1)$, and $(3,1,5)$ are image points (obtained from the pre-image points $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$). Since this is a linear transformation, we also know that the origin maps to itself, so $(0,0,0)$ is an image point. This means the vectors $\langle 2,1,4 \rangle$ and $\langle 1,0,1 \rangle$ are both in the image plane (you could also use the vector $\langle 3,1,5 \rangle$). Taking their cross product we get $\langle 1,2,-1 \rangle$ which must be perpendicular to the plane. Therefore the equation of the image plane is $x + 2y - z = d$. Since the image plane includes the origin, we can conclude that $d = 0$, so the equation is $x + 2y - z = 0$.

14. $X = (A - C)^{-1}(D - B)$

15. $\begin{bmatrix} \cos^2(\theta) - \sin^2(\theta) & 2\sin(\theta)\cos(\theta) \\ 2\sin(\theta)\cos(\theta) & \sin^2(\theta) - \cos^2(\theta) \end{bmatrix} = \begin{bmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$. We did $y = 2x$ already. Before you look at your notes, start by looking for the image point of $(1,0)$ and $(0,1)$. You'll have to use some knowledge of slopes and tangents. Don't stress on this. I won't ask anything this hard on the test.