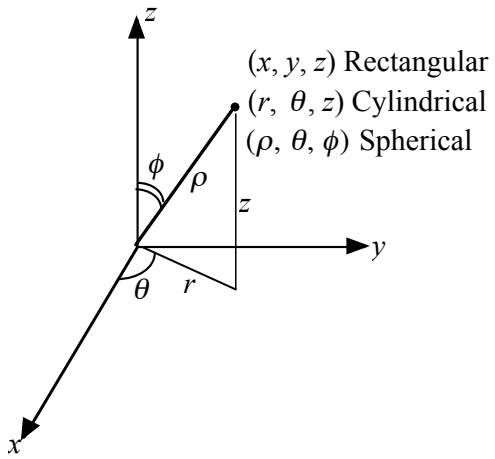
Honors Advanced Math Name:_____
More with Cylindrical and Spherical Coordinates Block: E or G 14 March 2014

Let us recall how to convert between rectangular and cylindrical or spherical coordinates.



Cylindrical: $(x, y, z) = (r\cos(\theta),\ r\sin(\theta), z)$

$$(r,\ \theta,\ \mathrm{z}) = \left(\sqrt{x^2 + y^2},\ \tan^{-1}\left(\frac{y}{x}\right), z\right)$$

Spherical: $(x, y, z) = (\rho\cos(\theta)\sin(\phi),\ \rho\sin(\theta)\sin(\phi),\ \rho\cos(\phi))$

$$(\rho,\ \theta,\ \phi) = \left(\sqrt{x^2 + y^2 + z^2},\ \tan^{-1}\left(\frac{y}{x}\right),\ \cos^{-1}\left(\frac{z}{\rho}\right)\right)$$

Converting points practice:

Cylindrical coordinates to rectangular

1. $\left(5,\ \frac{2\pi}{3},\ 7\right)$ $\left(-\frac{5}{2},\ \frac{5\sqrt{3}}{2},\ 7\right)$

2. $\left(15,\ \frac{5\pi}{4},\ 8\right)$ $\left(-\frac{15\sqrt{2}}{2},\ -\frac{15\sqrt{2}}{2},\ 8\right)$

3. $\left(2,\ \frac{\pi}{6},\ 9\right)$ $\left(\sqrt{3},\ 1,\ 9\right)$

Spherical coordinates to rectangular

4. $\left(5,\ \frac{2\pi}{3},\ \frac{5\pi}{4}\right)$ $\left(\frac{5\sqrt{2}}{4},\ -\frac{5\sqrt{6}}{4},\ -\frac{5\sqrt{2}}{2}\right)$

5. $\left(15,\ \frac{5\pi}{4},\ \frac{7\pi}{6}\right)$ $\left(\frac{15\sqrt{2}}{4},\ \frac{15\sqrt{2}}{4},\ -\frac{15\sqrt{3}}{2}\right)$

6. $\left(2,\ \frac{\pi}{6},\ \frac{4\pi}{3}\right)$ $\left(-\frac{3}{2},\ -\frac{\sqrt{3}}{2},\ -1\right)$

Rectangular coordinates to **a)** cylindrical and **b)** spherical

7. $(1, -1, 1)$

C: $\left(\sqrt{2},\ -\frac{\pi}{4}, 1\right)$

S: $\left(\sqrt{3},\ -\frac{\pi}{4}, 0.955\right)$

8. $\left(-\sqrt{2}, 1, 1\right)$

C: $\left(-\sqrt{3},\ -0.615, 1\right)$

S: $\left(2,\ -0.615,\ \frac{3\pi}{4}\right)$

9. $\left(-3,\ 4,\ -5\sqrt{3}\right)$

C: $\left(-5,\ -0.927,\ -5\sqrt{3}\right)$

S: $\boxed{\left(10,\ -0.927,\ \frac{5\pi}{6}\right)}$

Just as we spent one day converting coordinates between forms and then another day converting equations, we will spend this day converting cylindrical and spherical equations to rectangular forms.

Cylindrical Equations

$z = 0$ *xy* plane

$r = 5$ cylinder of radius 5 (so easy in cylindrical coordinates)

$\theta = \frac{\pi}{2}$ *yz* plane

$r = 1 - \sin(\theta)$ cardiod (heart) shaped tube

$r^2 + z^2 = 100$ $x^2 + y^2 + z^2 = 10^2$ sphere of radius 10

Spherical Equations

$\rho = 5$ sphere of radius 5 (so easy in spherical coordinates)

$\theta = \frac{\pi}{2}$ *yz* plane

$\phi = \frac{\pi}{6}$ double cone

$\rho \sin\phi = 2$ $r = 2$ or $x^2 + y^2 = 2^2$ cylinder of radius 2