

Honors Advanced Math
Practice Test Matrices

Name _____
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General instructions: Write a complete, fully explained solution to each problem, except where directions say otherwise. The quality of your responses will be a factor in grading. If you use your calculator for a major step, please briefly explain what you did on your calculator. Use exact answers whenever possible, unless the directions say otherwise.

1. Consider the linear transformation $T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $\det T < 0$.
 - a. Prove that the image of the unit square is a parallelogram. Include a sketch of the parallelogram.
 - b. Prove that the area of the image parallelogram is equal to the absolute value of the determinant of T .

2. a. Find the intersection of the following three planes. If the intersection is a point, find the coordinate of the point. If the intersection is a line, find the vector equation for the line. If the intersection is a plane, find the equation of the plane. If there is no intersection, demonstrate that this is the case.

$$3x + 2y - 6z = 11$$

$$-x + 4y + 16z = -13$$

$$2x + 5y + 7z = 0$$

- b. Find the partial fraction decomposition of $\frac{3x^2 + 34x - 62}{(x-2)(x-1)(x+4)}$. In other words, find values for A , B , and C such that $\frac{3x^2 + 34x - 62}{(x-2)(x-1)(x+4)} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{x+4}$.

3. Suppose that R_θ is a 2-by-2 transformation matrix that rotates about the origin by θ in the counter clockwise direction.

a. Show that the matrix of this rotation is $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$. Make sure your explanation is clear.

b. Suppose T is a matrix that first rotates by θ counterclockwise about the origin, and then reflects over the line $y = -x$. Find T . Carefully show how you got your answer.

c. What is the determinant of T ? Explain *geometrically*.

4. Consider the linear transformation $T = \begin{bmatrix} 1 & 1 & 5 \\ -1 & 2 & 4 \\ 4 & -1 & 5 \end{bmatrix}$.

a. Find the determinant of T . What does this tell you about the matrix transformation?

b. T maps all of 3-space to a single plane. Find the equation of this plane.

c. The null space of a transformation is the set of all points that map to the origin. Find the null space for T .

5. Find a 2 by 2 matrix for a single transformation that would reflect over the line $y = 3x$. You may give your matrix either with exact values or with decimal values rounded to the hundredths place. Show how you got this matrix.