Name _____ Honors Advanced Mathematics
March, 2013 Polar/complex Practice Test

1. a. Let P be the point in the plane corresponding to the complex number $7 - 4i$. Find two possible sets of polar coordinates (r_1, θ_1) and (r_2, θ_2) for P. Choose your answers so that $r_1 \neq r_2$ and $\theta_1 \neq \theta_2$.

b. Convert the rectangular equation $(x - 2)^2 + y^2 = 4$ into a polar equation. Simplify as much as possible, and solve for r.

2. Let $z = 2 \text{ cis } (\frac{\pi}{8}) = 2 [\cos (\frac{\pi}{8}) + i \sin(\frac{\pi}{8})]$ and let $w = 3 - 3i$.

a. Calculate zw in polar form using exact values (not decimal approximations).

b. Find positive whole number exponents n and k such that z^n is a negative real number and z^k is a positive real number.

3. **a.** Suppose $u = r[(\cos\theta) + i(\sin\theta)]$. Find and prove a formula for $\frac{1}{u}$.
(You are not allowed to assume that the DeMoivre's Theorem power formula has already been proved for negative exponents.)

b. Now suppose $u = (\cos\theta) + i(\sin\theta)$ [special case of part **a**, taking $r = 1$].
Consider u and $\frac{1}{u}$ as points in the complex plane. How are the locations of these two points related? Justify your answer.

4. Let $z = \frac{1}{2}\ \text{cis}\ \frac{3\pi}{4}$. Express answers to the following questions in polar form.

a. Calculate z^5.

b. Find the four other complex numbers whose 5th powers are the same as z^5.

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5. Consider the polynomial equation $x^3 + i = 0$.

a. Confirm that each of the following is a solution of the equation $x^3 + i = 0$.

$x = 1\ \text{cis}(210°)$	$x = 1\ \text{cis}(330°)$	$x = 1\ \text{cis}(-30°)$

b. Identify a solution of $x^3 + i = 0$ that does not equal any of the solutions listed in part **a**.

c. The *Fundamental Theorem of Algebra (FTA)* states "A degree n polynomial will have exactly n zeroes (counting multiplicity)." Explain how the results of parts **a** and **b** do not violate the *FTA*.

6. The three vertices of an equilateral triangle are given by the cube roots of $(\sqrt{3} - i)$ in the complex number system. Find decimal approximations of the (x, y) coordinates of the three points.

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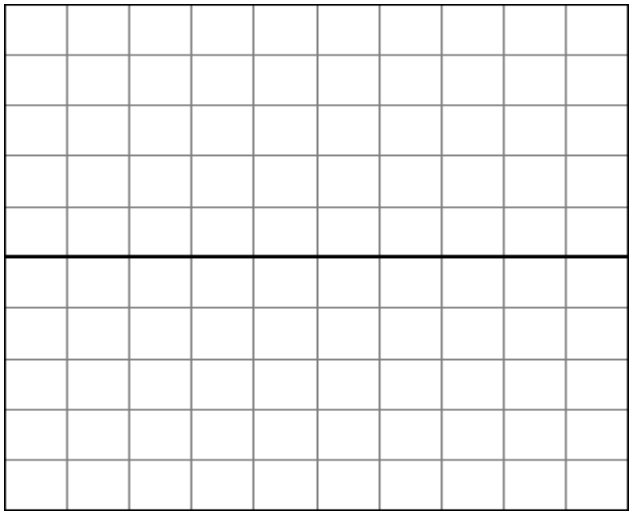
7. Without using your calculator, sketch an ordinary function graph and a polar graph for the polar equation $r = 2\sin(4\theta) + 3$

Ordinary function graph:



Polar Graph: