Name ___________________________________ Honors Advanced Mathematics
March, 2013 Polar/Complex Practice Test ANSWERS

1. a. $r^2 = 7^2 + (-4)^2$ so $r = \pm\sqrt{65}$. $\tan\theta = \frac{-4}{7}$ so $\theta = \tan^{-1}(\frac{4}{7}) + n\pi \approx -0.52 + n\pi$.
Must combine such that you get a point in the 4th quadrant; for example, could answer $(r_1, \theta_1) = (\sqrt{65}, -0.52)$ and $(r_2, \theta_2) = (-\sqrt{65}, 2.62)$.

b. $(r\cos\theta - 2)^2 + (r\sin\theta)^2 = 4$,
$r^2\cos^2\theta - 4r\cos\theta + 4 + r^2\sin^2\theta = 4$,
$r^2(\cos^2\theta + \sin^2\theta) = 4r\cos\theta$,
$r^2(1) = 4r\cos\theta$,
$r = 4\cos\theta$.

OR:

$x^2 - 4x + 4 + y^2 = 4$
$x^2 + y^2 = 4x$
$r^2 = 4r\cos\theta$
$r = 4\cos\theta$

2. Let $z = 2\operatorname{cis}(\frac{\pi}{8}) = 2[\cos(\frac{\pi}{8}) + i\sin(\frac{\pi}{8})]$ and let $w = 3 - 3i$.

a. $zw = 2\operatorname{cis}(\frac{\pi}{8}) \cdot 3\sqrt{2}\operatorname{cis}(-\frac{\pi}{4}) = 6\sqrt{2}\operatorname{cis}(-\frac{\pi}{8})$.

b. $n = 8$, $k = 16$ (from power formula: $n = 8$ makes the angle $-\pi$, $n = 16$ makes the angle -2π)

3. a. $\frac{1}{u} = \frac{1\operatorname{cis}0}{r\operatorname{cis}\theta} = \frac{1}{r}\operatorname{cis}(-\theta)$.

b. $u = 1\operatorname{cis}\theta$ and $\frac{1}{u} = 1\operatorname{cis}(-\theta)$ have the same radii but opposite angles, making the points reflected images of each other across the x-axis.

4. a. $z^5 = \frac{1}{32}\operatorname{cis}\frac{15\pi}{4}$.

b. $\frac{1}{2}\operatorname{cis}(\frac{3\pi}{4} + \frac{2\pi}{5}), \frac{1}{2}\operatorname{cis}(\frac{3\pi}{4} + \frac{4\pi}{5}), \frac{1}{2}\operatorname{cis}(\frac{3\pi}{4} + \frac{6\pi}{5}), \frac{1}{2}\operatorname{cis}(\frac{3\pi}{4} + \frac{8\pi}{5})$.

5. a. $x^3 + i = (1\operatorname{cis}(210°))^3 + i = 1\operatorname{cis}(630°) + i = -i + i = 0$, and similarly for the others.

b. $x = 1\operatorname{cis}(90°)$ or $x = i$.

c. Two of the zeros shown in part **a** actually coincide, because 330° and –30° are coterminal. Thus only 3 distinct zeros have been shown, not 4, so the FTA is not violated.

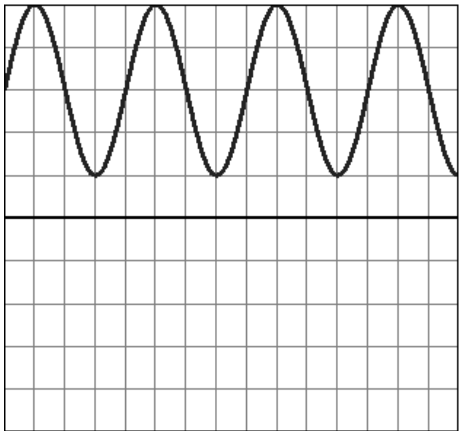
6. $\sqrt{3} - i = 2\operatorname{cis}(-\frac{\pi}{6})$ so its cube roots are:

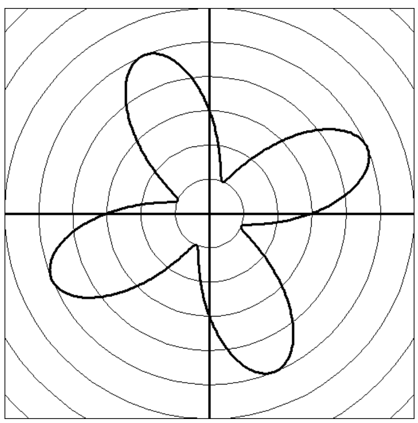
$\sqrt[3]{2}\operatorname{cis}(-\frac{\pi}{18}) \approx (1.24, -0.22)$

$\sqrt[3]{2}\operatorname{cis}(-\frac{\pi}{18} + \frac{2\pi}{3}) \approx (-0.43, 1.18)$

$\sqrt[3]{2}\operatorname{cis}(-\frac{\pi}{18} + \frac{4\pi}{3}) \approx (-0.81, -0.97)$

7. Graphs should have radius values ranging from 1 at the minimum to 5 at the maximum.





The following are points on the graphs:

r	θ
3	0
5	π/8
3	π/4
1	3π/8
3	π/2
5	5π/8
3	3π/4
1	7π/8

r	θ
3	π
5	9π/8
3	5π/4
1	11π/8
3	3π/2
5	13π/8
3	7π/4
1	15π/8