Honors Advanced Math

Name_____

Trigonometric Identities Practice Test

General instructions: *Write a complete, fully explained solution to each problem, except where directions say otherwise. The quality of your responses will be a factor in grading.*

1. a. Assuming only the sum and difference identities, prove the identity $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$.

b. Find the exact value of $\sin\left(\frac{13\pi}{12}\right)$. Your answer should not include any trigonometric functions.

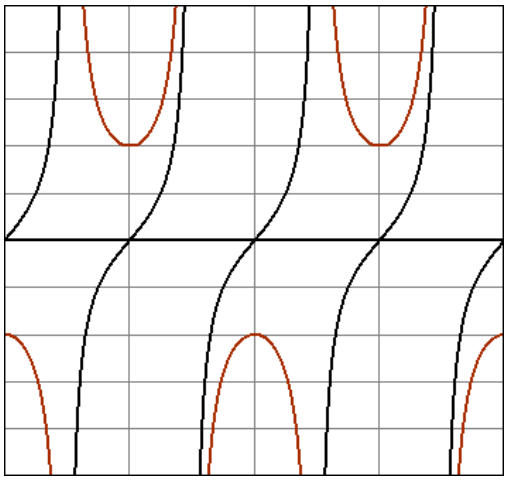
2. **a.** Prove $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. You may assume anything we have proven prior to this unit, but you may not assume any of the identities from the formula sheet.

b. Prove $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

3. **a.** Find all solutions to the equation $\cos(2x) + 5\cos x = 2$.

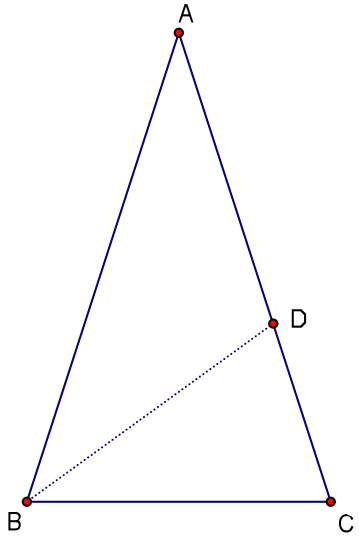
b. Prove the identity $\dfrac{1+\cos(2x)}{\cot(x)} = \sin(2x)$.

4. Consider the functions $f(x) = \tan(2x)$ and $g(x) = -2\sec(2x)$ which are graphed below. Though these functions don't intersect in the window shown, who is to say that they don't intersect outside of the window somewhere? Prove that the graphs of f and g *never* intersect.



5. In the given diagram, $\angle A = 36°$, $AB = AC = 1$, and segment BD bisects $\angle ABC$. Let $x = BC$.

a. Show that $\dfrac{x}{1-x} = \dfrac{1}{x}$.



b. Find the exact value of x.

c. Find the exact value of $\sin(18°)$.

Formula Sheet

Sum/Difference Identities

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double Angle Identities

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \begin{cases} \cos^2 u - \sin^2 u \\ 2\cos^2 u - 1 \\ 1 - 2\sin^2 u \end{cases}$$

$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

Power Reducing Identities

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Half Angle Identities

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$