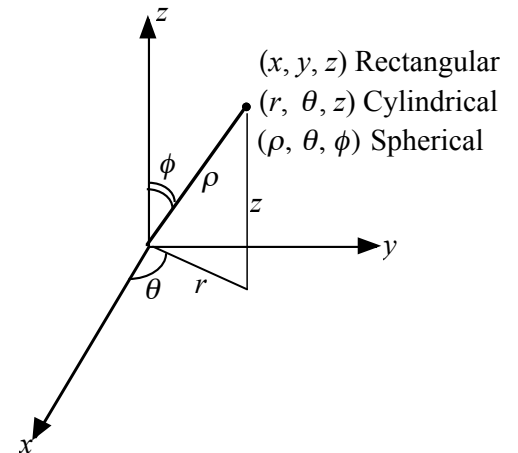


Just as adding a dimension added another parameter to our coordinates in the rectangular system, so too does adding a dimension to our polar system. There are two options. Use the (r, θ) from the xy plane and just say how far up to go in the z direction. This gives coordinates of the form (r, θ, z) . These are called *cylindrical* coordinates. The other option is to continue using a variable for the distance from the origin (this time the greek letter rho, ρ), and then use angle from the polar axis (the x -axis) and the z -axis. These are called *spherical* coordinates. Cylindrical coordinates are often used in engineering problems that deal with tubes (like submarines and airplane bodies). Spherical coordinates are common in physics and astronomy – situations where information that is centered around a point in space.



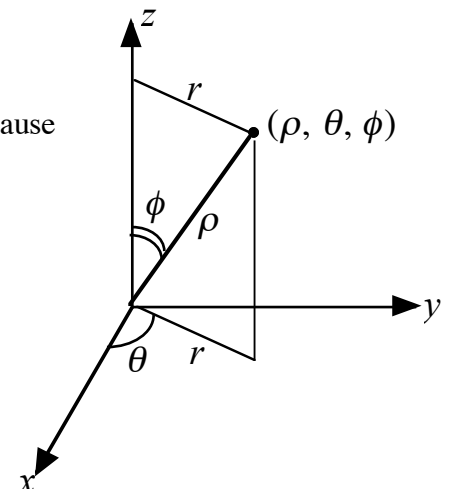
3D Problems in Cylindrical Coordinates

Converting from cylindrical to rectangular coordinates is easy because we use the same conversions that were used with 2D polar coordinates $(x, y, z) = (r\cos(\theta), r\sin(\theta), z)$. The following question asks about distances around the cylinder and through the silo. Use a prop if you are having trouble seeing this.

1. A farmer has a silo (a cylindrical silo). Given pairs of points on the silo, he needs to put a pipe connecting them through the silo and then tie a rope to each end of the pipe on the outside to keep the pipe in place. Calculate the lengths of the pipe and the rope for each of the following pairs of points.
 - a. $(6, 35^\circ, 38)$ and $(6, -325^\circ, 17)$
 - b. $(6, 45^\circ, 15)$ and $(6, 125^\circ, 15)$
 - c. $(6, 60^\circ, 12)$ and $(6, 240^\circ, 4)$

3D Problems in Spherical Coordinates

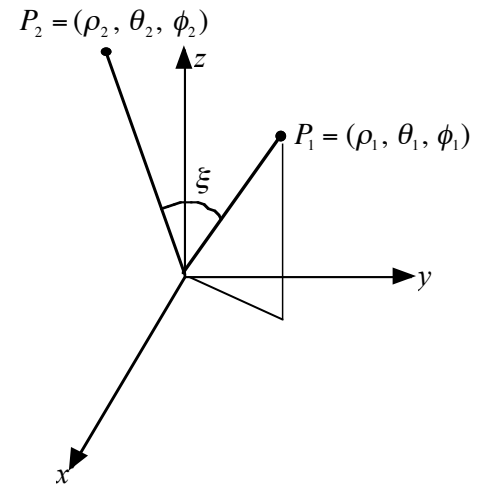
Converting from spherical to rectangular coordinates is not as easy because rho is not in any of the xyz planes. $(x, y, z) = (\rho\cos(\theta)\sin(\phi), \rho\sin(\theta)\sin(\phi), \rho\cos(\phi))$.



There are two considerations when trying to find the distance between points in space using spherical coordinates. If the two points are in spherically defined space (like the moons around Jupiter) then the distance between them is a straight line. If the two points are on a sphere (like two craters on the moon), the distance between them is an arc. In either case, the central angle is the most useful piece of information to find for solving the problem.

Let's derive a formula:

$$\cos(\xi) = \frac{\langle x_1, y_1, z_1 \rangle \cdot \langle x_2, y_2, z_2 \rangle}{|\overrightarrow{OP_1}| \cdot |\overrightarrow{OP_2}|} =$$



- The latitude and longitude of a point P in the Northern Hemisphere are related to spherical coordinates ρ, θ, ϕ as follows. We take the origin to be the center of the Earth and the positive z -axis to pass through the North Pole. The positive x -axis passes through the point where the prime meridian (the meridian through Greenwich, England) intersects the equator. Then the latitude of P is $\alpha = 90^\circ - \phi^\circ$ and the longitude is $\beta = 360^\circ - \theta^\circ$. Find the great-circle distance from Los Angeles (lat. 34.06°N , long. 118.25°W) to Montreal (lat. 45.50°N , long. 73.60°W). Take the radius of the Earth to be 3960 miles. (A *great circle* is the circle of intersection of a sphere and a plane through the center of the sphere.)
- When looking out at the night sky, from my viewpoint, Mars is 30° up, 15° to my left and 78 million kilometers away. Venus is 47° up and 25° to my right, and 42 million kilometers away. How far is Mars from Venus?

Polar Coordinate Extra Practice Problems

1. Convert the following polar equations to rectangular form and identify the graph. Support your answer with a graph.

a) $r = -2\csc(\theta)$ b) $r\csc(\theta) = 2$ c) $r = 1 + 2\cos(3\theta)$ d) $r = -3\sin(\theta)$

2. Given the following rectangular coordinates, find at least two versions of the corresponding polar coordinates.

a) $(1, 2)$ b) $(-2, 3)$ c) $(-3, -4)$ d) $(4, -5)$

3. Graph, by hand, $r = 4\cos(\theta) - 4\sin(\theta)$ on the polar grid below:

