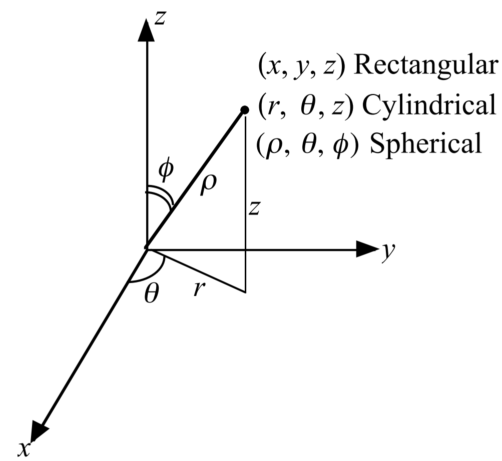
Honors Advanced Math — Name:______________________

Polar Coordinates 2D and 3D — 12 March 2014

Just as adding a dimension added another parameter to our coordinates in the rectangular system, so too does adding a dimension to our polar system. There are two options. Use the (r, θ) from the *xy* plane and just say how far up to go in the *z* direction. This gives coordinates of the form (r, θ, z). These are called *cylindrical* coordinates. The other option is to continue using a variable for the distance from the origin (this time the greek letter rho, ρ), and then use angle from the polar axis (the *x*-axis) and the *z*-axis. These are called *spherical* coordinates. Cylindrical coordinates are often used in engineering problems that deal with tubes (like submarines and airplane bodies). Spherical coordinates are common in physics and astonomy – situations where information is centered around a point in space.



3D Problems in Cylindrical Coordinates

Converting from cylindrical to rectangular coordinates is easy because we use the same conversions that were used with 2D polar coordinates $(x, y, z) = (r\cos(\theta),\ r\sin(\theta), z)$. The following question asks about distances <u>around</u> the cylinder and <u>through</u> the cylinder. Use a prop if you are having trouble seeing this.

1. A farmer has a silo (a cylindrical silo). Given pairs of points on the silo, he needs to put a pipe connecting them through the silo and then tie a rope to each end of the pipe on the outside to keep the pipe in place. Calculate the lengths of the pipe and the rope for each of the following pairs of points.

a. $(6,\ 35°,\ 38)$ and $(6,\ -325°,\ 17)$ pipe through = rope around = 21

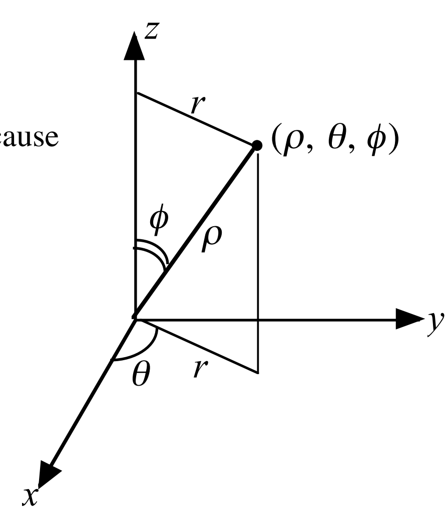
b. $(6,\ 45°,\ 15)$ and $(6,\ 125°,\ 15)$ pipe through = 7.713, rope around = 8.377

c. $(6,\ 60°,\ 12)$ and $(6,\ 240°,\ 4)$ pipe through = 14.422, rope around = 20.477

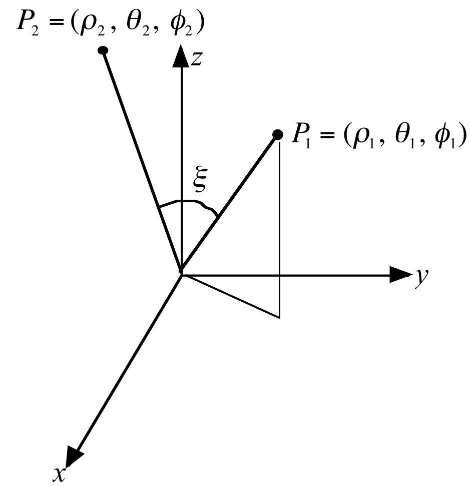
3D Problems in Spherical Coordinates

Converting from spherical to rectangular coordinates is not as easy because rho is not in any of the *xyz* planes.

$$(x, y, z) = (\rho\cos(\theta)\sin(\phi),\ \rho\sin(\theta)\sin(\phi),\ \rho\cos(\phi)).$$



There are two considerations when trying to find the distance between points in space using spherical coordinates. If the two points are <u>in</u> spherically defined space (like the moons around Jupiter) then the distance between them is a straight line. If the two points are <u>on</u> a sphere (like two craters on the moon), the distance between them is an arc. In either case, the central angle is a useful piece of information to find. Think about how you could find the distance between the two points and how that could be used to find the angle between them.



2. The latitude and longitude of a point P in the Northern Hemisphere are related to spherical coordinates ρ, θ, ϕ as follows. We take the origin to be the center of the Earth and the positive z-axis to pass through the North Pole. The positive x-axis passes through the point where the prime meridian (the meridian through Greenwich, England) intersects the equator. Then the latitude of P is $\alpha = 90° - \phi°$ and the longitude is $\beta = 360° - \theta°$. Find the great-circle distance from Los Angeles (lat. 34.06°N, long. 118.25°W) to Montreal (lat. 45.50°N, long. 73.60°W). Take the radius of the Earth to be 3960 miles. (A *great circle* is the circle of intersection of a sphere and a plane through the center of the sphere.)

LA = (3960, 241.75°, 55.94°) Montreal = (3960, 286.4°, 44.5°)

arc length distance <u>on</u> the Earth = 2464.176 miles

3. When looking out at the night sky, from my viewpoint, Mars is 30° up, 15° to my left and 78 million kilometers away. Venus is 47° up and 25° to my right, and 42 million kilometers away. How far is Mars from Venus?

Mars = (78 million, 15°, 60°) Venus = (42 million, 335°, 43°)

distance = 85.947 million miles

Polar Coordinate Extra Practice Problems

1. Convert the following polar equations to rectangular form and identify the graph. Support your answer with a graph.

a) $r = -2\csc(\theta)$

$y = -2$

horizontal line

b) $r\csc(\theta) = 2$

$x^2 + y^2 = 2y$

circle, $r = 1$

centered at (0, 1)

c) $r = 1 + 2\cos(3\theta)$

$(x^2 + y^2)^2 = (x^2 + y^2)^{1.5} + 2x(x^2 - y^2)$

rose curve

3 small, 3 large

d) $r = -3\sin(\theta)$

$x^2 + y^2 = -3y$

circle, $r = 1.5$

centered at (0, –1.5)

2. Given the following rectangular coordinates, find at least two versions of the corresponding polar coordinates.

a) (1, 2)

$\left(\sqrt{5},\ 1.107\right)$

$\left(-\sqrt{5},\ 4.248\right)$

b) (-2, 3)

$\left(\sqrt{13},\ 2.159\right)$

$\left(-\sqrt{13},\ 5.3\right)$

c) (-3, -4)

$\left(5,\ 4.069\right)$

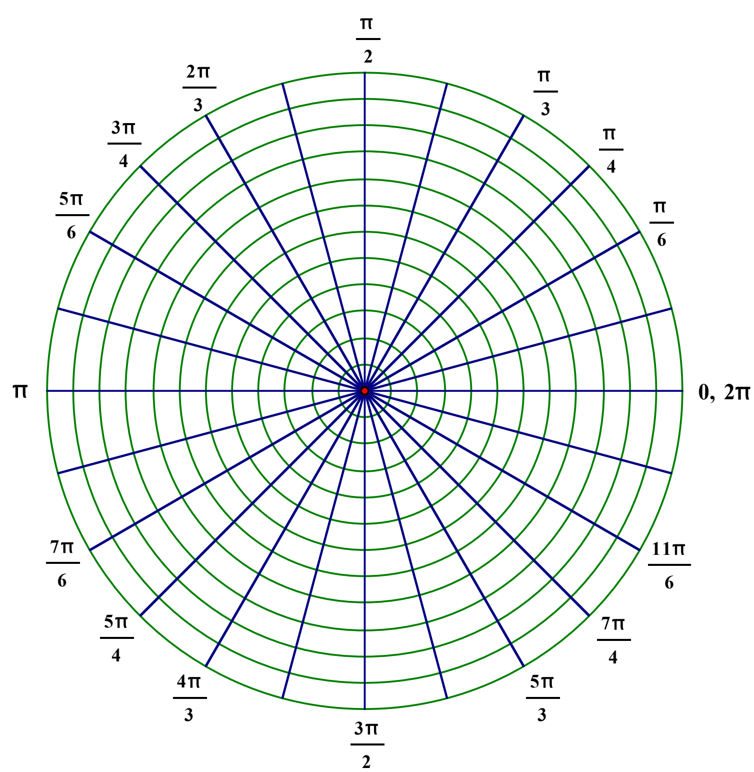
$\left(-5,\ 0.927\right)$

d) (4, -5)

$\left(\sqrt{41},\ 5.387\right)$

$\left(-\sqrt{41},\ 2.2455\right)$

3. Graph, by hand, $r = 4\cos(\theta) - 4\sin(\theta)$ on the polar grid below:



4. Consider the polar equation $r = a \cdot \cos(n\theta)$ when n is an odd integer.

a) Prove that the graph is symmetric about the x-axis.

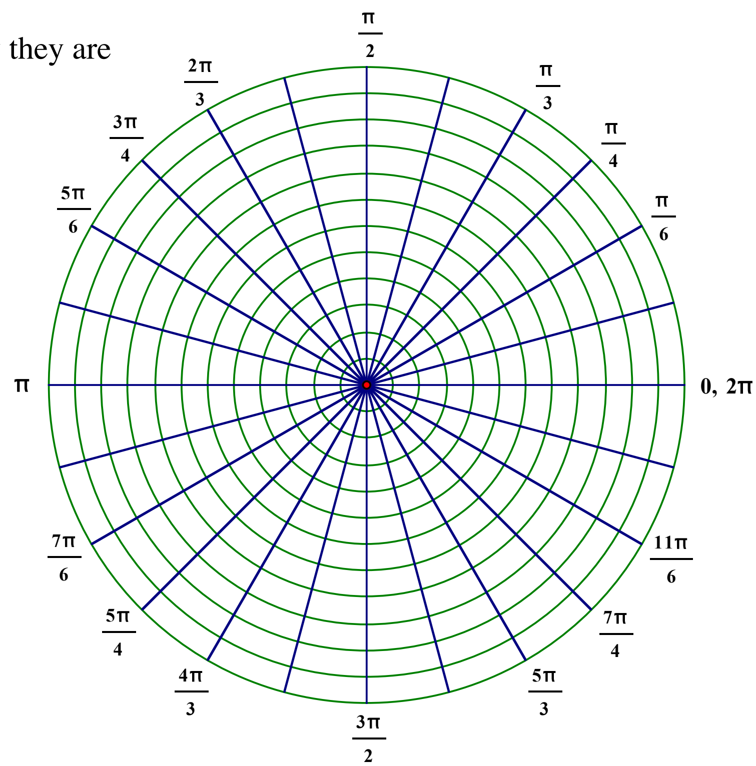
b) Prove that the graph is not symmetric about the y-axis.

c) Prove that the graph is not symmetric about the origin.

d) Prove that the maximum value of r is $|a|$.

5. Graph each polar equation and describe how they are related to each other.

a) $r_1 = 1 + 3\sin(3\theta)$



b) $r_2 = 1 - 3\sin(3\theta)$