Honors Advanced Math
Trig Identity Review Packet

Name: **ANSWER KEY**

For the Trig Identitity test you should be able to:

- Derive any of the angle sum, double angle, half angle or power reducing identities
- Solve equations by using the identities to simplify the equations
- Prove identities (other than the basics)
- Use the identities to solve problems

The practice test online includes a reference sheet with several identities. You will not be given a sheet like this for the real exam. You must know the basic identities and be able to derive the others from them.

1. Use the cofunction identities and odd-even identities to prove that $\sin(\pi - x) = \sin(x)$.

$$\sin(\pi - x) = \sin\left(\frac{\pi}{2} - \left(x - \frac{\pi}{2}\right)\right) = \cos\left(x - \frac{\pi}{2}\right) = \cos\left(-\left(\frac{\pi}{2} - x\right)\right) = \cos\left(\frac{\pi}{2} - x\right) = \sin(x)$$

2. Use the cofunction identities and odd-even identities to prove that $\cos(\pi - x) = -\cos(x)$.

$$\cos(\pi - x) = \cos\left(\frac{\pi}{2} - \left(x - \frac{\pi}{2}\right)\right) = \sin\left(x - \frac{\pi}{2}\right) = \sin\left(-\left(\frac{\pi}{2} - x\right)\right) = -\sin\left(\frac{\pi}{2} - x\right) = -\cos(x)$$

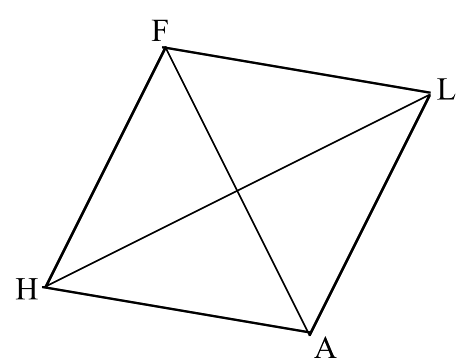
3. Use the identity in problem **1** to prove that in any triangle ABC, $\sin(A + B) = \sin(C)$.

In a triangle, $A + B + C = \pi$ so $C = \pi - (A + B)$ so $\sin(C) = \sin(\pi - (A + B)) = \sin(A + B)$

4. Use the identities in problems **1** and **2** to find an identity for simplifying $\tan(\pi - x)$.

$$\tan(\pi - x) = \frac{\sin(\pi - x)}{\cos(\pi - x)} = \frac{\sin(x)}{-\cos(x)} = -\tan(x)$$

5. A rhombus is a quadrilateral with equal sides. The diagonals of a rhombus bisect the angles of the rhombus and are perpendicular bisectors of each other. Let $\angle ALF = \alpha$ and $\angle LHA = \beta$ Each side is of length s.



a) Show that $\cos\left(\frac{\alpha}{2}\right) = \frac{HL}{2s}$ and $\sin\left(\frac{\alpha}{2}\right) = \frac{FA}{2s}$

b) Show that $\sin(\alpha) = \frac{HL \cdot FA}{2s^2}$ (Start with the area of ΔLAF, calculated from the trig triangle area and from the geometric triangle area)

c) Show that the area of the rhombus is $s^2 \sin(2\beta)$ ($\angle AHF = 2\beta$ and the area of the rhombus is the area of the two congruent triangles.)

d) Use the fact that the diagonals are perpendicular to show that its area is $2s^2 \sin(\beta) \cdot \cos(\beta)$

Area $HALF$ = 2*(0.5 * (FA * 0.5 HL)) and use the results from part a

e) Use your answers from **(c)** and **(d)** to obtain a formula for $\sin(2\beta)$ set the areas equal and solve.

6. In ΔABC, the measure of $\angle B$ is twice the measure of $\angle C$.

a. Use the law of sines to show that $b = 2c \cos C$

Start with $\frac{c}{\sin C} = \frac{b}{\sin B} = \frac{b}{2\sin C \cos C}$ and solve for c.

b. Use the law of cosines to show that $b^2 = c(a + c)$

Start with $c^2 = a^2 + b^2 - 2ab\cos C$ and replace $\cos C$ using the result in part a. Factor out a b^2/c from the last two terms, subtract a^2 from c^2 and factor the difference of squares. You should now see where to go to finish.

7. Solve each of the following for $0 \le x < 2\pi$ (except **e** and **f**) using trigonometric identities:

a. $\sin x \cos x = \frac{1}{2}$ $\quad x = \frac{\pi}{4}, \frac{5\pi}{4}$

b. $\cos 2x = 5\sin^2 x - \cos^2 x$ $\quad x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

c. $\tan 2x + \tan x = 0$ $\quad x = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

d. $\sin 2x \cdot \sec x + 2\cos x = 0$ $\quad x = \frac{3\pi}{4}, \frac{7\pi}{4}$

e. $\cos^{-1} 2x = \sin^{-1} x$ $\quad x = \frac{\sqrt{5}}{5}$

f. $\tan^{-1} 2x = \sin^{-1} x$ $\quad x = 0, \pm\frac{\sqrt{3}}{2}$

8. Here are letter name definitions for some trigonometric values.

$A = \sin(0.1\pi)$	$C = \sin(0.2\pi)$	$E = \cos(0.27\pi)$
$B = \cos(0.1\pi)$	$D = \cos(0.23\pi)$	$F = \tan(0.3\pi)$

Express each of the following trigonometric values in terms of A, B, C, D, E, and/or F. For example, if $\csc(0.1\pi)$ were given as a problem, the answer would be $\csc(0.1\pi) = \frac{1}{A}$. You may not use trigonometric inverses in your answers.

a. $\cot(0.1\pi) =$ B / A

b. $\cos(0.3\pi) =$ $\cos(0.5\pi - 0.2\pi) = \sin(0.2\pi) = C$

c. $\cos(0.73\pi) =$ $\cos(\pi - 0.27\pi) = -\cos(0.27\pi) = -E$

d. $\sin(-0.3\pi) = -\sin(0.3\pi) = -\tan(0.3\pi) * \cos(0.3\pi) = -F * C$

e. $\tan(1.4\pi) = \tan(\pi + 0.4\pi) = \tan(0.4\pi) = \tan(0.5\pi - 0.1\pi) = \cot(0.1\pi) = B / A$

f. $\cos(0.5 \sin^{-1}(\cos(0.3\pi))) = \cos(0.5 \sin^{-1}(C)) = \cos(0.5 * 0.2\pi) = \cos(0.1\pi) = B$